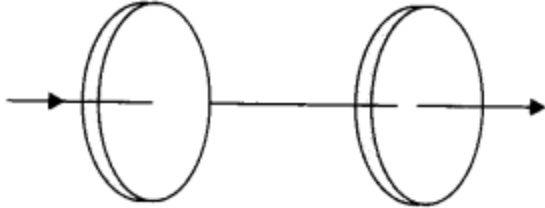# 8. Electromagnetic Waves

Question 1.

Figure shows a capacitor made of two circular plates each of radius 12 cm, and separated by 5.0 mm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.

(a) Calculate the capacitance and the rate of change of potential difference between the plates.

(b) Obtain the displacement current across the plates.

(c) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.

Solution:
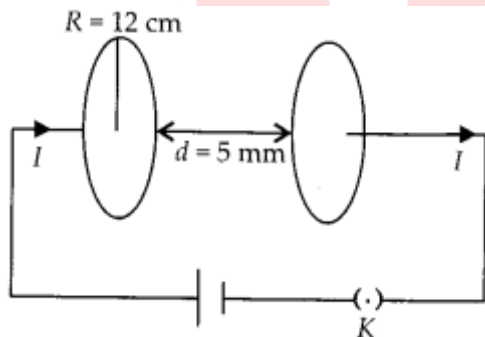
(a) Capacitance of the parallel plate capacitor

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times \pi \times (12 \times 10^{-2})^2}{5 \times 10^{-3}}$$

$$C = 80.1 \times 10^{-12} \text{ F} = 80.1 \text{ pF}$$

We know $Q = CV$

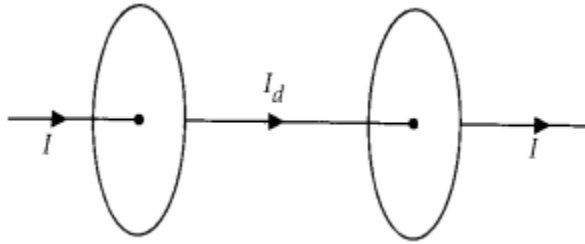$$I = \frac{dQ}{dt} = C \frac{dV}{dt}$$

So, $\dfrac{dV}{dt} = \dfrac{I}{C} = \dfrac{0.15}{80.1 \times 10^{-12}} = 1.875 \times 10^9 \text{ V s}^{-1}$

(b) The magnitude of displacement current is equal to conduction current. $\therefore I_d = I = $

0.15 A

(c) Yes, Kirchhoff's first law is very much applicable to each plate of capacitor as $l_d = I$. So current is continuous and constant across each plate.

Question 2.

A parallel plate capacitor as shown in figure made of circular plates each of radius R = 6.0 cm has a capacitance C = 100 pF. The capacitor is connected to a 230 V ac supply with a (angular) frequency of 300 rad s^{-1}.

(a) What is the rms value of the conduction current?

(b) Is the conduction current equal to the displacement current?

(c) Determine the amplitude of 6 at a point 3.0 cm from the axis between the plates.
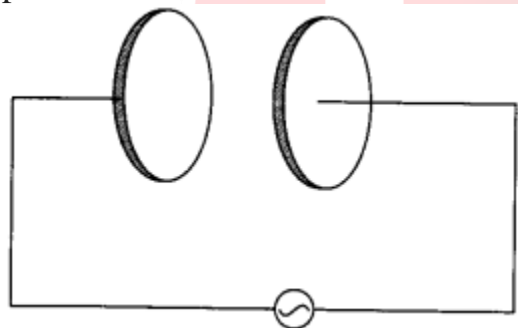
Solution:

(a) Capacity of capacitor C = 100 pF

$$\text{Capacitive reactance } X_C = \frac{1}{\omega C}$$

$$= \frac{1}{300 \times 100 \times 10^{-12}}$$

$$\therefore \quad X_C = \frac{10^8}{3} \, \Omega$$
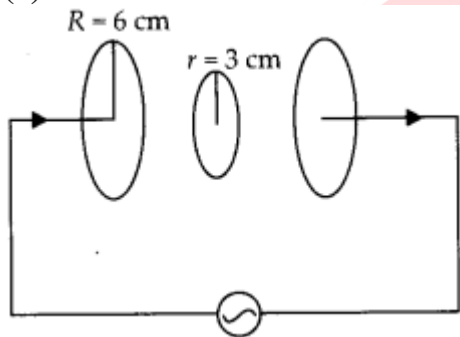
If I_{rms} is the rms value of conduction current

$$I_{rms} = \frac{E_{rms}}{X_C} = 230 \times 3 \times 10^{-8}$$

$$= 690 \times 10^{-8} = 6.9 \, \mu A$$

(b) Yes, the conduction current in wires is always equal to displacement current within plates.

(c)

To find magnetic field B of a point 3 cm from the axis within plates, let us assume a loop of radius 3 cm with center on axis. Now modified Ampere's Law.

$$\int \vec{B} \cdot \overline{dl} = \mu_0 \left[\frac{I}{\pi R^2} \pi r^2 \right]$$

$$B \times 2\pi r = \frac{\mu_0 I r^2}{R^2}$$

For amplitude of magnetic field, we require

$$I_0 = I_{rms} \sqrt{2} = 6.9\sqrt{2} \, \mu A$$

So, $\quad B_0 = \frac{\mu_0}{2\pi} \times \frac{I_0 r}{R^2}$

$$= 2 \times 10^{-7} \times \frac{6.9\sqrt{2} \times 10^{-6} \times 3 \times 10^{-2}}{(6 \times 10^{-2})^2}$$

$$B_0 = 1.63 \times 10^{-11} \, T$$

Question 3.

What physical quantity is same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 A and radio waves of wavelength 500 m?

Solution:
Though they all have different wavelengths and frequencies, but they have same speed i.e., speed of light 3×10^8 m s^{-1} in vacuum.

Question 4.
A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of its electric and magnetic field vectors? If the frequency of the wave is 30 MHz, what is its wavelength?
Solution:
In electromagnetic wave, the electric field vector E and magnetic field vector B show their variations perpendicular to the direction of propagation of wave as well as perpendicular to each other. As the electromagnetic wave is travelling along z-direction, hence E and B show their variation in x-y plane.

$$\text{Wavelength, } \lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8 \text{.m/s}}{30 \times 10^6 \text{ s}^{-1}} = 10 \text{ m.}$$

Question 5.
A radio can tune in to any station in the 7.5 MHz to 12 MHz band. What is the corresponding wavelength band?
Solution:
Maximum wavelength in the band is for lowest frequency

$$c = \upsilon_{min} \lambda_{max}$$

$$\lambda_{max} = \frac{c}{\upsilon_{min}} = \frac{3 \times 10^8}{7.5 \times 10^6} = 40 \text{ m}$$

Minimum wavelength in the band is for highest frequency

$$c = \upsilon_{max} \lambda_{min}$$

$$\lambda_{min} = \frac{c}{\upsilon_{max}} = \frac{3 \times 10^8}{12 \times 10^6} = 25 \text{ m}$$

Question 6.
A charged particle oscillates about its mean equilibrium position with a frequency of 109 Hz. What is the frequency of electromagnetic wave produced by the oscillator?
Solution:
The frequency of electromagnetic wave produced will be same as the frequency of oscillation of charged particles i.e., 109 Hz.

Question 7.

The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave?

Solution:

We know the relation, $\dfrac{E_0}{B_0} = c$

$E_0 = B_0 c = 510 \times 10^{-9} \times 3 \times 10^8$

Peak value of $E_0 = 1530 \times 10^{-1} = 153$ N C^{-1} electric field.

Question 8.

Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N C^{-1} and that its frequency is $u = 50.0$ MHz. **(a)** Determine B_0, to, k, and X. **(b)** Find expressions for E and B.

Solution:

(a) By relation, $E_0 B_0 = c$

Peak value of magnetic field

$$B_0 = \frac{E_0}{c} = \frac{120}{3 \times 10^8} = 40 \times 10^{-8} \text{ T}$$

Angular frequency, $\omega = 2\pi v$

$$= 100\pi \times 10^6 \text{ Hz.}$$

Speed of light $c = v\lambda$

Wavelength, $\lambda = \dfrac{c}{v} = \dfrac{3 \times 10^8}{50 \times 10^6}$, $\lambda = 6$ m

Propagation constant,

$$k = \frac{2\pi}{\lambda} = \frac{\pi}{3} \text{ m}^{-1} = 1.05 \text{ m}^{-1}$$

(b) Let the wave is propagating along A'-direction, electric field E is along y –

direction and magnetic field B along z-direction.

$$\vec{E}_y = E_0 \sin(kx - \omega t)\hat{j}$$

$$\vec{E}_y = 120 \sin\left(\frac{\pi}{3}x - 100\pi \times 10^6 t\right)\hat{j}$$

$$\vec{B}_z = B_0 \sin(kx - \omega t)\hat{k}$$

$$= 40 \times 10^{-8} \sin\left(\frac{\pi}{3}x - \pi \times 10^8 t\right)\hat{k}$$

Question 9.

The terminology of different parts of the electromagnetic spectrum is given in the NCERT text. Use the formula $E = h\upsilon$ and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation?

Solution:

The photon of given energy are released during transition between energy levels in the atom and the emitted photon energy is equal to difference of energies of energy levels, among which the transition has taken place. For example photon energy for $\lambda = 1$ m can be calculated

$$E = h\upsilon = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1} \text{ J}$$

$$E = \frac{19.88 \times 10^{-26}}{1.6 \times 10^{-19}} \text{eV} = 12.425 \times 10^{-7} \text{ eV}$$

So, for emission of radio waves the energy difference between energy levels should be 12.425×10^{-7} eV. Similarly, photon energy for other wavelengths can also be calculated

Type of radiation	Wavelength of radiation	Photon energy
1. Radio waves	1 m	12.425×10^{-7} eV
2. Micro waves	3×10^{-3} m	4.14×10^{-4} eV
3. Infrared waves	3×10^{-5} m	4.14×10^{-2} eV
4. Visible waves	5×10^{-7} m	2.485 eV
5. Ultraviolet rays	3×10^{-7} m	4.14 eV
6. X-rays	10^{-10} m	12.425×10^3 eV
7. γ rays	10^{-12} m	12.425×10^5 eV

So, for emission of y -rays the energy difference among energy levels should be of the order of MeV, where for visible radiation it should be of the order of eV.

Question 10.
In a plane electromagnetic wave, the electric field oscillates sinusoidally at a frequency of 2.0×10^{10} Hz and amplitude 48 Vm^{-1}.
(a) What is the wavelength of the wave?
(b) What is the amplitude of the oscillating magnetic field?
(c) Show that the average energy density of the E field equals the average energy density of the B field, [$c = 3 \times 10^8$ m s^{-1}.]
Solution:
(a) Wavelength of electromagnetic wave is

$$\lambda = \frac{c}{\upsilon} = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$$

(b) Amplitude of magnetic field, $E_0 B_0 = c$

$$B_0 = \frac{E_0}{c} = \frac{48}{3 \times 10^8} = 16 \times 10^{-8} \text{ T}$$

(c) Energy density as electric field,

$$u_E = \frac{1}{2} \varepsilon_0 E^2$$

Here, $\frac{E}{B} = c$

So, $u_E = \frac{1}{2} \varepsilon_0 c^2 B^2$

where speed of electromagnetic wave,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

So $\quad u_E = \frac{1}{2} \frac{\varepsilon_0}{\mu_0 \varepsilon_0} B^2$

$$u_E = \frac{B^2}{2\mu_0} = u_B$$

[Energy density as magnetic field]

Question 11.
Suppose that the electric field part of an electromagnetic wave in vacuum is £ = [(3.1 N/C)] cos[(1.8 rad/m) y + (5.4 x 10^8 rad/s)f)] i.

(a) What is the direction of propagation?
(b) What is the wavelength λ?
(c) What is the frequency υ?
(d) What is the amplitude of the magnetic field part of the wave?
(e) Write an expression for the magnetic field part of the wave.
Solution:
(a) Electromagnetic wave is propagating in -y direction.
(b) Propagation constant, $k = 2\pi\lambda = 1.8$

$$\lambda = \frac{2\pi}{1.8} = \frac{\pi}{0.9} \text{ m} = 3.49 \text{ m}$$

(c) angular frequency, $\omega = 2\pi\upsilon = 5.4 \times 10^8$

$$\upsilon = \frac{5.4}{2 \times 3.14} \times 10^8 = 86 \times 10^6 \text{ Hz} = 86 \text{ M Hz}$$

(d) Amplitude of magnetic field pat, $E_0 B_0 = c$

$$\text{So, } B_0 = \frac{E_0}{c} = \frac{3.1}{3 \times 18^8} = 1.03 \times 10^{-8} \text{ T}$$

$$= 10.3 \times 10^{-9} \text{ T} = 10.3 \text{ nT}$$

(e) Expression for magnetic field pat of wave

$$\vec{B}_z = B_0 \cos(ky + \omega t)\,\hat{k}$$
$$\vec{B}_z = 1.03 \times 10^{-8}\cos(1.8y + 5.4 \times 10^6 t)\,\hat{k}$$

Question 12.

About 5% of the power of a 100 W light bulb is converted to visible radiation.
What is the average intensity of visible radiation
(a) at a distance of 1 m from the bulb?
(b) at a distance of 10 m?
Assume that the radiation is emitted is o tropically and neglect reflection.
Solution:
The bulb as a point source, radiate light in all direction uniformly and it is given that only 5% of power is converted to visible radiations.
(a) Let us assume a sphere of radius 1 m. Surface area $A = 4\pi r^2 = 4\pi\,(1)^2 = 4\pi$
m^2 Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \times 0.05}{4\pi} = 0.4 \text{ W m}^{-2}$$

(b) Let us assume a sphere of 10 m radius. Surface area $A = 4\pi^2 = 4K\,(10)^2 = 400\pi$
m^2 Intensity on the sphere

$$I = \frac{\text{Power}}{\text{Area}} = \frac{100 \times 0.05}{400\pi} = 0.004 \text{ W m}^{-2}$$

Question 13.
Use the formula $\lambda_m T = 0.29$ cm K to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain tell you?

Solution:
A black body at very high temperature produce a continuous spectrum. Using Wien's displacement law we can calculate the wavelength corresponding to maximum intensity of radiation emitted Required absolute temperature

$$\lambda_m T = 0.29 \text{ cm K} \quad \text{or} \quad T = \frac{0.29 \times 10^{-2}}{\lambda_m} \text{ K}$$

Using above formula, the temperature of black body required for various wavelength is calculated.

	Type of radiation	Wave-length of radiation	Tem-perature required
1	Radio waves	1 m	29×10^{-4} K
2.	Micro waves	3×10^{-3} m	1 K
3.	Infrared waves	3×10^{-5} m	10^2 K
4.	Visible waves	5×10^{-7} m	6×10^3 K
5.	Ultraviolet rays	3×10^{-7} m	10^4 K
6.	X-rays	10^{-10} m	29×10^6 K
7.	γ-rays	10^{-12} m	29×10^8 K

To produce electromagnetic radiations of different wavelength, we need temperature ranges. To produce visible ' radiation of X = 5×10^{-7} m, we need to have source at temperature of 6000 K. A source at lower temperature will produce this wavelength but not with maximum intensity.

Question 14.
Given below are some famous numbers associated with electromagnetic radiations in different contexts in Physics. State the part of the electromagnetic spectrum to which each belongs.
(a) 21 cm [wavelength emitted by atomic 'hydrogen in interstellar space].
(b) 1057 MHz [frequency of radiation arising from two close energy levels in hydrogen; known as Lamb shift].
(c) 2.7 K [temperature associated with the isotropic radiation filling all space-

thought to be a relic of the 'big-bang' origin of the universe].

(d) 5890 A – 5896 A [double lines of sodium]

(e) 14.4 keV [energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mossbauer spectroscopy)].

Solution:

(a) 21 cm wavelength corresponds to radio waves, at nearly 1445 MHz.

(b) Electromagnetic wave of 1057 MHz also corresponds to radio waves.

(c) Using Wien's Law the wavelength of the radiation mostly emitted at given temperature of heavenly bodies can be calculated

$$\lambda_m T = b = 0.29 \text{ cm K}$$

$$\text{or} \quad \lambda_m = b/T = \frac{0.29}{2.7} = 0.11 \text{ cm}$$

This wavelength corresponds to microwave region of electromagnetic waves.

(d) This wavelength corresponds to visible region (yellow) of electromagnetic waves.

(e) The frequency of radiation can be calculated

$$E = h\upsilon$$

$$\upsilon = \frac{E}{h} = \frac{14.4 \times 10^3 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \text{ Hz}$$

$$= 3.49 \times 10^{18} \text{ Hz}$$

This frequency corresponds to X-ray region of electromagnetic waves.

Question 15.

Answer the following questions:

(a) Long distance radio broadcasts use short-wave bands. Why?

(b) It is necessary to use satellites for long distance TV transmission. Why?

(c) Optical and radio telescopes are built on the ground but X-ray astronomy is possible only from satellites orbiting the earth. Why?

(d) The small ozone layer on top of the stratosphere is crucial for human survival. Why?

(e) If the earth did not have an atmosphere, would its average surface temperature be higher or lower than what it is now?

(f) Some scientists have predicted that a global nuclear war on the earth would be followed by a severe 'nuclear winter' with a devastating effect on life on earth. What might be the basis of this prediction?

Solution:

(a) Electromagnetic waves in frequency range of shortwave band reflect from ionosphere where lower frequency radio waves i.e., medium waves are absorbed. So, short waves are suitable for long distance radio broadcast.

(b) Very high frequency (VHF) and ultra high frequency (UHF) electromagnetic waves used in T.V transmission after frequency modulation do not reflect from ionosphere and can only be transmitted straight from antenna. So, the range is highly limited. In order to send the transmission for long distances, satellite is used.

(c) Atmosphere absorb X-rays, while visible and radio waves can penetrate through it. Hence optical telescope can work on ground but X-ray astronomical telescopes only work above atmosphere, hence installed on the satellite orbiting around earth.

(d) The thin ozone layer absorbs harmful ultraviolet radiations, y-rays and cosmic radiations. All these high energy radiations can cause damage to the living cells. Thus the thin ozone layer on top of the stratosphere is crucial for human survival.

(e) In the absence of atmosphere, the day temperature can rise to many fold and temperature at night will be much below 0°C. Although the average temperature will be reduced in the absence of green house effect.

(f) The thick clouds produced by global nuclear war will prevent solar radiation to reach down across the globe. This would cause severe 'nuclear winter'.